2. Spatial analytic geometry

2.1. Triangle ABC is given by vertices $A = [-1, 2, 3]$, $B = [1, 1, 1]$ and $C = [0, 0, 5]$. Prove, that this triangle is right-angled and determine the magnitude of angle β.

2.2. Calculate the unit vector $\mathbf{c}$, that is perpendicular to vectors $\mathbf{a} = (2, -1, 1)$ and $\mathbf{b} = (1, 2, -1)$.

2.3. Calculate the area of triangle ABC with vertices $A = [2, 3, 4]$, $B = [-1, 2, -3]$ and $C = [5, 4, -2]$.

2.4. Decide, whether points $A = [1, 2, -1]$, $B = [0, 1, 5]$, $C = [-1, 2, 1]$ and $D = [2, 1, 3]$ lie in the same plane. If so, write down parametric, general and intercept equations of the plane.

2.5. Calculate the angle between planes ρ and σ, where the plane ρ is given by points $A = [0, 0, 0]$, $B = [1, 1, 1]$ and $C = [3, 2, 1]$ and the plane σ by points $K = [0, 0, 0]$, $L = [1, 1, 1]$ and $M = [3, 1, 2]$.

2.6. Given the cube $ABCDEFGH$, calculate the angle between planes ABC and ACF.

2.7. Write down the equation of plane ρ, that passes through points $A = [1, 3, 2]$ and $B = [2, 2, 1]$ and is perpendicular to plane $\phi : 2x - y - z + 4 = 0$.

2.8. Write down the equation of plane ρ, that passes through point $A = [4, -7, 5]$ and is parallel to plane (xz).

2.9. Write down the equation of plane ρ, that passes through point $A = [4, -7, 5]$ and intersects x-, y- and z-axes at the same distance from the origin.

2.10. Determine the distance between parallel planes $\rho : 2x - y + 2z + 5 = 0$ and $\sigma : 2x - y + 2z - 7 = 0$.

2.11. Determine the equation of plane ρ, that contains the line of intersection of planes $3x - 2y - 2z = -1$ and $x - 3y - z + 2 = 0$ and passes through point $A = [3, 1, 3]$.

2.12. Write down parametric equations of median passing through vertex C of triangle ABC, where $A = [3, 6, -7]$, $B = [-5, 2, 3]$ and $C = [4, -7, -2]$.

2.13. Determine the angle between lines $p : \begin{cases} x(t) = 5 + 2t \\ y(t) = 2 - t \\ z(t) = 7 + t \end{cases}$ and $q : \begin{cases} x + 3y + z + 2 = 0 \\ x - y - 3z + 2 = 0 \end{cases}$.

2.14. Determine the distance of point $A = [1, -1, -2]$ from line $p : \begin{cases} x(t) = -3 + 3t \\ y(t) = -2 + 2t \\ z(t) = 8 + 2t \end{cases}$.

2.15. Lines p and q are given. In case of intersecting lines, determine coordinates of point of intersection. Otherwise, determine their shortest distance. Lines are given by: $p : X = [3, 4, -1] + t(1, 1, -1)$ and q passes through points $B = [-6, -5, 1]$ and $C = [0, 7, -2]$.

2.16. The plane ϕ passes through point $A = [0, -3, 0]$ and is perpendicular to planes $\alpha : x + 2y + 3z = 5$ and $\beta : 3x - 5y + 4z = 12$. Write down its general equation.

2.17. Determine the angle between line p passing through points $A = [2, 0, 3]$ and $B = [2, 2, 2]$ and plane $\alpha : 3z - y + 10 = 0$.

Quadrics

2.18. Write down the equation of sphere κ with centre in point $[3, 0, -2]$ passing through point $[3, 0, 0]$.

2.19. Write down the equation of cylinder of revolution ω which axis of revolution is identical to x-axis and base centre is $[3, 0, 0]$. The cylinder passes through point $[3, 0, -3]$.

2.20. Write down the equation of tangent planes of sphere $\kappa : (x-2)^2+(y+1)^2+(z-3)^2 = 6$ at points, where the line $p : X = [1, 0, 1+t(1, -1, 2)]$ intersects the sphere.

2.21. Determine the curve of intersection of hyperbolic paraboloid $\psi : z = 4x^2 - y^2$ and plane:

- $y = 2$
- $z = 1$
- $z = 0$.

2.22. Determine coordinates of point of intersection of quadric $\omega : x^2 + y^2 - z^2 = 19$ and line $p : X = [0, 2, 1] + t(1, -1, 1)$.

2.23. Determine coordinates of point of intersection of quadric $\delta : x^2 + y^2 - z^2 = 2$ and line $p : X = [1, 1, 0] + t(1, -1, \sqrt{2})$.

2.24. The quadric $\lambda : x^2 + 4x + y^2 - 2y + 4z^2 + 8z + 5 = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.25. The quadric $\gamma : x^2 - 2x + y^2 - 4y + z^2 + 2z + 6 = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.26. The quadric $\rho : 4x^2 - 16x - y^2 - 6y + 4z^2 + 8z + 7 = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.27. The quadric $\kappa : x^2 - 4x - y^2 + 6y - z^2 + 2z - 7 = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.28. The quadric $\beta : x^2 + y^2 - z^2 + 2z - 1 = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.29. The quadric $\alpha : x^2 - 4x - y^2 - 2y = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.30. The quadric $\epsilon : x^2 - 6x + y^2 + 4y - z + 15 = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.31. The quadric $\varphi : x^2 - 2x - y^2 - z + 4 = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

2.32. The quadric $\sigma : x^2 + x - y^2 - y = 0$ is given. Determine the type of quadric, its centre or vertex and semiaxes. Sketch it in technical isometry and draw its projection onto coordinate planes.

Regions in E_3:

2.33. Draw the region, bounded by surfaces $z = \sqrt{x^2 + y^2}$, $z = 2 - x^2 - y^2$. Draw its projection onto planes (xy) and (yz), too. Determine its section with plane $z = 0$ and plane $z = 2$.

2.34. Draw the region, bounded by surfaces $3z = x^2 + y^2$, $x^2 + -y^2 = z^2$. Determine its section with plane (xy) and (yz) and draw all.

2.35. Draw the region, bounded by surfaces $x = 0$, $y = 0$, $z = 0$, $2x + 3y + 6z = 6$. Determine its section with plane (xy) and (yz) and draw all.

2.36. Draw the region, bounded by surfaces $x = 0$, $y = 0$, $z = 0$, $2x + 3z = 6$ and $y = 4$. Determine its section with plane (xy) and (yz) and draw all.

2.37. Draw the solid arisen from the intersection of surface-bounded spaces $z \leq -\sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 \leq 1$. Determine boundary surfaces and boundary curves. Draw projections onto planes (xy) and (yz), too.